e-learning mathematics*

Sebastià Xambó Descamps† (*moderator*)
Hyman Bass, Gilda Bolaños Evia, Ruedi Seiler‡,
Mika Seppälä§ (*panelists*)

Abstract. In addition to the current state of knowledge about the learning of mathematics and its aims in today's society, the main purpose of this paper is discussing ways of improving the process of learning, and especially, in that regard, the role of e-learning technologies. We chart the situation of e-learning mathematics as of December, 2005, including distance-learning or open university courses, and then we consider a number of areas where e-learning is likely to develop. Finally, we assess the impact of e-learning on the role of the new educators in mathematics.

Mathematics Subject Classification (2000). Primary: 97-xx, 97Uxx; Secondary: 00-xx.

Keywords. Online material, distance learning, e-learning, metadata.

Presentation

by *Sebastià Xambó Descamps*

Following a suggestion of the Executive Committee (EC) of ICM2006 that came forth in the Fall of 2004, this panel has been promoted by the Conference of Spanish Mathematics' Deans [1].

After having formally accepted the invitation on December 16, 2004, the CDM Executive Committee discussed possible topics, until "e-Learning Mathematics" (eLM) was chosen and approved by both the CDM and the EC of ICM2006. Names to be invited as panelists were also decided, and it is a great satisfaction, and an honour, to be able to say that all accepted. On behalf of the CDM, my sincerest thanks to all.

If e-learning is learning by means of systems built on current computer and communications technologies, then the main interest of eLM is on what advantages e-learning can offer in the case of mathematics.

The main reason for choosing eLM is that the accelerated evolution of the e-Learning field is having, and will most likely continue to have, a major worldwide impact

*Panel promoted by the Spanish Conference of Mathematics' Deans.

†Partially supported by the European e-Content project "Web Advanced Learning Technologies" (WebALT), Contract Number EDC-22253.

‡Thanks for the support by the Bundesminister für Bildung und Forschung.

§Partially supported by the European e-Content project "Web Advanced Learning Technologies" (WebALT), Contract Number EDC-22253.

Proceedings of the International Congress
of Mathematicians, Madrid, Spain, 2006

on many aspects of the teaching-learning systems, at all levels, while offering, at the same time, new opportunities to professional mathematicians and to existing or new institutions, as for example in life-long learning. It is thus a topic that should greatly interest not only mathematicians in all walks of life, but also academic and political authorities everywhere.

This is why we imagined that the panel could aim at describing the situation of eLM as of 2006, outlining the most likely trends of its evolution in the next few years, indicating what the strongest impacts (positive or negative) in the mathematics teaching-learning systems will be, and charting the sorts of opportunities that will arise.

We are of course aware that such aims can only be attained by the panel in very broad terms, although this should be enough to bring forward a generally useful picture. For those wanting to have more detailed views, the references provided by the panelists should be a valuable resource to continue a journey that by all evidence has no return. For example, the articles in the recent book [2] will quite likely be serviceable to a wide range of readers seeking to know more about e-learning in general.

Let me continue with a few general remarks on learning, teaching and e-learning.

Mathematics, or mathematics knowledge, is a vast universe (let me call it M). It has many smaller interelated universes, of which we have a dim glimpse in the standard classifications.

Because of the increasing number of research mathematicians, and the availability of ever more sophisticated computational and communication tools, M has undergone an extraordinary growth, and all indications are that this trend will continue in the coming years. To a large extend this blooming is explained because M is both a source of deep beauty and the only precision method we have for modelling the physical universe.

In any case, the number of university students required to take mathematics courses is globally increasing, but at the same time the number of professional mathematicians that seek a teaching position is most likely decreasing, as there are, on one side, ever newer job profiles, and, on the other side, the number of students in mathematics degrees is decreasing in most countries. Moreover, in the last decade a steady decline in the mathematical skills of the students beginning higher education has been reported (see, for example, [3]).

Can eLM help to face this situation in a more positive mood?

The expectations created by e-learning are certainly high, at all levels, and we may wonder how much of it is going to be true, and up to what point can it help in the case of mathematics.

The reasons behind the high expectations on e-learning stem from well-known characteristics of the e-learning systems:

- In principle, access is possible from anywhere and at any time, thus making possible flexible (even just-for-me) and just-in-time courses of learning.

- The teacher can also be anywhere and do most of his teaching job at any time (preparing materials or following-up and coaching his students).
- It allows for synchronous activities of a teacher and a group (at an agreed time), but again without restriction on the location of the people involved, and, what is more, with the possibility of addressing a much larger audience than a conventional class.
- Assessment can be automated to a large extend and final grading can be integrated seamlessly into the institution's information system.
- The learning materials and experiences can be richer in many ways, and they can be easily maintained and updated (as compared to preparing, say, a new edition of a paper book).
- There are also indications that it may induce deeper understanding and stronger retention.

So the main question is how can we harness all that potential for improving the quantity and quality of the learning of mathematics. Since there are many levels that we ought to consider, and many variations in each level, we cannot expect a universal recipe. And even if we restrict ourselves to a very particular situation, say remedial mathematics for freshman in engineering schools or mathematical modules for prospective secondary school teachers, we cannot expect a formula that would satisfy everybody.

A sensible starting point is just looking at people, groups and institutions that are leading the way in one direction or another. This is the idea behind the purpose and composition of this panel. Since it is not feasible, and perhaps not even desirable, under the circumstances, to have a comprehensive survey of eLM, the best alternative is having experts in a few areas that have a major bearing on what eLM is and can be, and on how it is evolving. Before going into their reports, let me briefly introduce them.

Hyman Bass

Hyman Bass is Roger Lyndon Collegiate Professor of Mathematics and Professor of Mathematics Education at the University of Michigan. A graduate of Princeton, Dr. Bass earned his Ph.D. from the University of Chicago under Irving Kaplansky. He has had visiting appointments at sixteen different universities in ten countries. The many honors and prizes that Dr. Bass has received include the Cole Prize in algebra. He is an elected member of the American Academy of Arts and Sciences and the National Academy of Arts and Sciences, and the Third World Academy of Sciences, and was elected Fellow of the American Association for the Advancement of Science. He is former president of the American Mathematical Society and current president of ICMI. He has been both a Sloan and Guggenheim Fellow. Dr. Bass has published eighty-six papers in mathematics and seventeen in mathematics education.

Gilda Bolaños

Dr. Bolaños is a certified teacher and trainer in the didactical techniques of Problem Based Learning (PBL) and Project Oriented Learning (POL). She is the author of several certified Blackboard courses. With classroom technologies based on Maple and Minitab, she has worked extensively on problems and materials for her online courses.

Ruedi Seiler

Full Professor for Mathematics at the Technische Universität Berlin, Ruedi Seiler's main fields of interest are Mathematical Physics, Quantum-Hall Systems, Information Theory, Data Compression, and E-Math: Teaching, Learning, Research. Member of the Research Center "Mathematics for Key Technologies" (DFG), and of the Executive Committee of the International Association of Mathematical Physics (IAMP), his most recent undertakings, culminating an extensive experience in organizing events and participating in projects, are MUMIE and MOSES. More specifically, he is leading, since 2001, the project "Multimedial Mathematical Education for Engineers", a project developed in Cooperation between the Berlin University of Technology, the Munich University of Technology, the Aachen University of Technology and the University Potsdam (funded by the German Federal Ministry of Education and Research within the programme "New Media in Education"), and, within the program "Notebook-University" of the German Federal Ministry of Education and Research, he is co-manager, since 2002, of the TU Berlin project "MOSES – Mobile Service for Students".

Mika Seppälä

Dr. Seppälä is Professor of Mathematics at Florida State University and Professor of Computer Aided Mathematics at the University of Helsinki. He was the Co-ordinator of the HCM network "Editing and Computing" (1995–1996) which initiated the development that lead to the MathML and OpenMath languages allowing the inclusion of mathematical formulae on the web pages in a meaningful way. He is currently the Secretary of the OpenMath Society, and the co-ordinator of the eContent Project "Web Advanced Learning Technologies" (WebALT). The main goal of the WebALT Project is to use MathML and OpenMath to create tools and content for multilingual on-line mathematics. Seppälä was the President of the Finnish Mathematical Society for the period 1992–1996.

Sebastià Xambó Descamps

Full Professor of Information and Coding Theory at the Universitat Politècnica de Catalunya (UPC, Barcelona, Spain), and former Full Professor of Algebra at the Departamento de Algebra of the Universidad Complutense of Madrid (1989–1993), is serving as Dean of the "Facultat de Matemàtiques i Estadística" of the UPC. Member of the EU eContent Project "Web Advanced Learning Technologies". In the period 1994-2000 led the team that developed the mathematical engine of Wiris ([4], [5]) and authored the e-book [6]. Cofounder of Maths for More ([7]). Has served as President of the Societat Catalana de Matemàtiques (1995–2002) and of the Executive Committee of the 3rd European Congress of Mathematics (Barcelona, 2000), and as Vicerector of Information and Documentation Systems of the UPC (1998–2002). Since the Fall of 2004 he serves as President of the Spanish Conference of Mathematics' Deans.

References

[1] http://www.usc.es/mate/cdm/.

[2] *Integrated E-learning. Implications for Pedagogy, Technology and Organization*. Edited by Wim Jochems, Jeroen van Merriënboer and Rob Koper. RoutledgeFalmer, 2004.

[3] http://www.engsc.ac.uk/index.asp.

[4] Eixarch, R., Marquès, D., Xambó, S., WIRIS: An Internet platform for the teaching and learning of mathematics in large educational communities. *Contributions to Sience* **2** (2) (2002), 269–276.

[5] Eixarch, R., Marquès, D., Xambó, S., Report on the positive effects for an educational community of having universal Internet access to a mathematical computational system. In preparation.

[6] Xambó, S., *Block Error-Correcting Codes: A Computational Primer*. Springer-Verlag, 2003. Digital version at http://www.wiris.com/cc/.

[7] http://www.mathsformore.com/.

The instructional potential of digital technologies

by *Hyman Bass*

Educational uses of technology. Digital technology continues to rapidly transform all aspects of life and work, even (and perhaps all the more so) in the developing world. It is designed, and presumed, to bring great benefit and empowerment to its users, as well as profit to its developers. Yet, as it opens new and even unanticipated possibilities, it poses as many problems as it solves, some new, and some techno-versions of classical problems, all of them important and interesting. And technology, for its novelty and glamorous aspirations, is greedy for our attention, liking to take center stage in every arena it enters.

Education, and mathematics education in particular, is the context in which this panel is examining these transformations. I find it helpful here to distinguish three broad kinds of roles that technology can play in mathematics education. They are of course not disjoint.

I. ***Transmission:*** Use of technology (web, video conferencing, etc.) to transmit, perhaps interactively, instruction and/or instructional materials that are conceptually of a traditional genre – lectures, demonstrations, problem sets, assessments, etc. These are the kinds of uses that fundamentally support distance learning, for example.

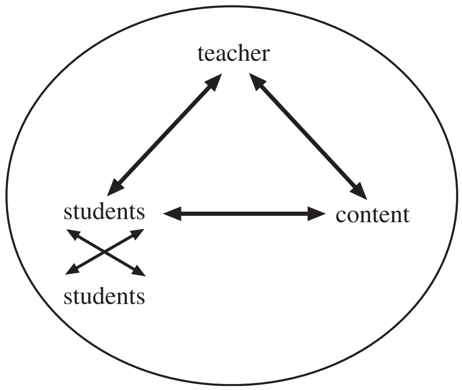
II. ***Power, speed, and visualization:*** Use of technology to carry out quickly and more accurately and completely, mathematical processes of a traditional nature – perform large or complex calculations, solve equations, approximate integrals, exhibit function graphs, study effects of variation of parameters, produce vivid and accurate images of geometric figures, etc.

III. ***New ways to explore the (mathematical and experiential) universes:*** Use of technology to do things we have never previously been able to do. Such capability affects mathematics itself, not just mathematics education. Examples include the study of long-term evolution of dynamical systems, and the images of fractal geometry that emerge there from. (This had an effect on dynamics comparable with that of the telescope in astronomy and the microscope in biology.) Software development gave life to the field of computational complexity, with its applications to coding and cryptography. Mathematical modeling and computer simulation supports a virtually empirical study of physical systems and designs. Dynamic geometry offers unprecedented opportunities to visually explore and analyze geometric structures, and to produce evocative imagery of dimensions three and four (using time). Computer algebra systems furnish unprecedented resources for solving equations. Much of this new technological power is now within reach of many students, and this raises possibilities of thereby expanding the horizons of the mathematics curriculum.

At a pragmatic level, technology thus offers resources to address two fundamental challenges of contemporary education – distance and demographics. Distance because many learners in need are physically remote from the sources of quality instruction and materials. Gilda Bolaños offers us an excellent survey of diverse modes of distance learning formats. Demographics because class sizes, particularly in introductory level mathematics courses, are too large to afford adequate instructor attention to individual student learning. (Bounding class sizes is often done at the cost of using instructors of highly variable quality.) In this case, technology affords various interactive formats for student work and assessment. These include the "virtual laboratories" described by Ruedi Seiler, and the interactive online materials (lectures, automatically graded homework, etc.) discussed by Mika Seppälä.

But independently of these practical needs, technology also offers possibilities for improving mathematics instruction itself. And the fundamental questions about the quality of teaching and learning do not recede when the instruction is mediated by technology; they only change their form.

Instruction. By "instruction" I mean the dynamic interaction among teacher, content, and students. I rely here on the "instructional triangle" that Cohen and Ball use to depict the set of interactions that they call "instruction" (Cohen and Ball, 1999).



Viewed in this way, instruction can go wrong in some simple but profound ways, for its quality depends on the relations among all of these three elements. When they misconnect, students' opportunities for learning are impaired. For example, if a teacher is not able to make the content accessible to students, framing it in ways that are incomprehensible to them, the chances that they may misunderstand are great. If students' interpretations of a task are different from the teacher's or the textbook author's intentions, then their work may be misrouted or take the work in unhelpful directions.

It may seem slightly strange, in the context of this panel, to propose the above representation of instruction. For, if you think about it, most descriptions of instructional uses of technology appear to reside exclusively on the bottom edge of the instructional triangle, absent the teacher. A tacit premise of some of this thinking is that somehow, the technology, with its interactive features, actually substitutes for the teacher, or renders the teacher obsolete, except perhaps as a manager of the environment. The viability of this view is a deep and important question, one that I shall not enter here except to make a couple of observations. One is that, in the most successful models of distance learning, it was found to be essential to have a tutor or facilitator available at the remote sites of reception of the materials, to respond to the many questions and requests that students would have, and that were not adequately responded by the technology environment. In addition, it was found to be important to have real time online questioning of the primary source available at certain times. In other words, prepared and transmitted material alone no more teaches a learner than does a textbook, unmediated by a teacher. The other comment is that interactive technology formats can at best provide well-prepared instructional materials and tasks, and respond to the student productions and questions that the software developers have anticipated and for which they have programmed responses. There are many domains of procedural learning and performance where this can be somewhat successful, though the software, no more than a skilled teacher, cannot completely predict and prepare for all of what students may come up with. Moreover this uncertainty is all the greater once one enters into territory that is less procedural and involves more conceptual reasoning and problem solving.

In what follows, I identify five persistent problems of mathematics instruction and discuss ways in which technology can be deployed to address these. How these are actually used, however, would affect the degree to which they were helpful, so for each case, I point out its possible pitfalls.

1. Making mathematically accurate and pedagogically skillful diagrams. One problem faced by mathematics teachers at all levels is how to make clear and accurate diagrams that make the essential mathematical ideas plain to learners, and how to do so in ways that are manipulable for mathematical reasoning. Doing this by hand is often no easy task, whether the sketch is of slices of an ellipsoid in calculus, or sixteenths of a rectangle in fifth grade. Mathematical accuracy is one dimension of the challenge; featuring is a second - that is, making the instructionally key features visible to learners. In addition, instructors must manage these challenges fluently, using class time effectively. An instructor who can make diagrams accurately and helpfully, but who must use 10 minutes of class time to do so, loses effectiveness. Diagrams are also used for a variety of purposes: explorationally, to investigate what happens if certain elements are allowed to vary, or presentationally, to demonstrate an idea, an explanation, or a solution. This means, sometimes, the need for dynamics - translations, rotations, rescaling, variation of parameters. Often diagrams must be made in ways that map clearly to algebraic or numerical representations. Drawing software, or other design tools, can help. Important is the capacity to produce carefully-scaled diagrams, with the capacity for color or shading, and to be able to move elements of a diagram. Its use must be fast and flexible, helpful both for carefully designed lectures and for improvisation on the fly, in response to a student's question. Such software or tools can provide significant support for the use of diagrams in class, by both students and instructor. Making such software accessible to students increases their capacity for individual explorations and preparation for contributions in class. Students can quickly put their diagrams up for others' inspection, or support a point in class, in ways that are difficult to do when students go to the board to generate representations by hand. Using software tools to support the visual dimensions of mathematical work in instruction can significantly alter a major dimension of instruction and do so in ways that are mathematically accurate, pedagogically useful, and sensitive to the real-time challenges of classroom instruction where class periods are finite and time is a critical resource.

Software tools to support the making of diagrams can create problems, too. For example, if the tools are rigid or interfere with the purposes for making diagrams, or cannot be manipulated as desired, the representations may not be as useful as needed. Another problem may be that the use of such tools inhibits students from developing personal skills of appraisal and construction. If the tools quickly make correct diagrams, students may not develop a critical eye with which to inspect them. If they never have to make a diagram themselves, they may remain entirely dependent on the software and not develop independent capacities for drawing.

2. Making records of class work and using them cumulatively across time. A second pervasive problem of mathematics instruction can be seen in the overflowing blackboards full of work and the slippery sheets of transparencies filled with notation and sketches, generated in class, and that vanish into weak memory when class ends. The record of class work (not just text or prepared materials), whether lecture, discussion, or exploration, is an important product of instruction. Under ordinary circumstances, this product vanishes and is thus unavailable for study or future reference, use, or modification. So acute is this problem that, too often, even during a single class, such work is erased (in the case of chalkboards) or slid away (as in transparencies). The work of that single class period is weakened for not being able to secure its place in evolution of ideas in the course. Moreover it is not available for students who may have missed a class.

When the work done in class is created or preserved in digital form, an archive of the mathematical progress of the class can become a resource for ongoing learning. It can then be easily accessed and transmitted remotely to others. Doing it "live" in class requires skill and dexterity on the part of the instructor. Making records of classwork afterwards (i.e., photographing the board with a digital camera) is easier but possibly less manipulable for subsequent class work. Important, too, is that everyone who needs to access these records can work on a common platform or that the format will work reliably across platforms.

3. Alignment between classes and textbook. Instructors, perhaps in response to student ideas or productions, may choose to depart from the text - in topic treatment or sequencing, or even topic coverage, and in the design of student activities and tasks. If the instructor creates these variations and alternative paths in electronic form, then a new text is created based on the instructor's design. This affords students access to the substance and course of the lessons. This gives license to flexible and innovative instruction, by affording the means to do so without disadvantaging students through disconnection from a text to be perused and revisited over time.

4. Ease of access to the instructor between classes. In the developed world, it is hard to imagine university instructors who do not maintain email (and web) connection with their students. This has made much more fluent and elastic the traditional functions of "office hours." Most student questions can be handled expeditiously, in timely fashion (though asynchronously), by email (perhaps with attachments), thus greatly reducing the need for face-to-face meetings, with their scheduling difficulties. And, as with the discussion above, these exchanges can contribute significantly to the record of the student's work and progress. When appropriate, an exchange between one student and the instructor can easily be made available to other students, thus changing an individual "office hour" into a group discussion. Pitfalls can exist with electronic communications, of course. Misunderstanding is frequent when communication is restricted to text, without gesture, intonation, and the ability to demonstrate or show.

5. The repetitive nature of individual outside-of-class sessions. One feature of traditional office hours, or help sessions, is that they tend to be repetitive, processing over and over again the same questions and assistance with each new student or group of students. When such assistance is administered electronically, and it is seen to be germane to the interests of the whole class, it is an easy matter to copy the whole class, or perhaps selected individuals, on such exchanges. This puts to collective profit the considerable instructional investment made in one student, or group of students, and everyone gains, not least the instructor. An important consideration here is sensitivity to privacy issues and confidentiality. In particular, making individual student communications requires prior consent.

Conclusion. Technology continues to transform all aspects of our lives and work. It is already difficult to imagine how we once functioned without email and the web. We are still at the early stages of trying to understand and design the best uses of technology for mathematics instruction. I have pointed to some promising uses of technology to address some endemic problems of even traditional instruction. I have also tried to signal that the fundamental problem of developing quality teaching does not disappear just because instruction is mediated in technological environments.

References

Cohen, D. K., and Ball, D. L., *Instruction, capacity, and improvement*. (CPRE Research Report No. RR-043). University of Pennsylvania, Consortium for Policy Research in Education, Philadelphia, PA, 1999.

Distance learning today

by *Gilda Bolaños Evia*

The definition of distance learning has been modified over time, and today we have a variety of definitions. We will adopt the definition of Greenberg, in [Greenberg98], where contemporary distance learning is defined as “a planned teaching/learning experience that uses a wide spectrum of technologies to reach learners at a distance and is designed to encourage learner interaction and certification of learning”.

In this section we will discuss the effects of some of the technologies used in distance learning education on mathematics and its effects on student’s knowledge.

Video taped lectures. Since the introduction of videos to instruct students on different areas, many studies have been conducted to determine the effectiveness of these methods. Some examples are [Beare 89], [Moore 96], [Russell 97], and [Pflieger 61]. On all of these studies the conclusion is that there is no significant difference on the achievement of students on video classes and regular classes. A three-year study involving 200,000 students and 800 public schools states:

> "... whereas most comparisons showed no significant differences, 119 were significant in favor of TV-taught students, and 44 in favor of conventionally taught students." [Pflieger 61].

We have to observe that on these studies the quality of the taught material was the same for video students and traditional students. Due to the lack of availability of similar studies for Latin America, we asked some professors and authorities that have been part of the VIBAS (video high school system) about the effectiveness of the system. In general they think that there is a significant difference in favor of traditional education, but this difference is not because of the video system, but mainly because of quality of materials and lack of availability of tutors. Moore and Kearsky converge to the same opinion in [Moore 96]. They also estimate that the difference is bigger in mathematics and physics. Coordinators of mathematics departments in public universities in Guanajuato State, Mexico, have noticed that students coming from video systems have a higher probability to fail its first math courses. They argue that their math knowledge is lower compared with regular students.

In the opinion of these authorities video taped lectures will tend to disappear, but not in the near future, at least for underdeveloped countries, because it is one of the cheapest forms to deliver distance education. They will be replaced by technologies as videoconferences.

At some universities video taped lectures are used inside the classroom for very specific concepts within the syllabus to present an expert opinion. Teachers at the Instituto Tecnológico y de Estudios Superiores de Monterrey (ITESM) highly recommend this instrument for advanced courses and also to present interesting and attractive applications on elementary courses.

Video conference. Video conference has been used within higher education for more than a decade. Video conferencing is highly used for teaching sessions, teachers training, seminars and research. At many universities video conference is used as a tool to bring into the class an international experimented and recognized teacher to a large number of students. From the experience at ITESM, has been determined that the success of a video conference class depends on such factors as:

a) Quality of sound, images and degree of interaction.

b) Compatibility of the equipment with ingoing and outgoing signal places.

c) Availability and quality of material presented in the video conference.

d) Quick response to students questions.

e) A tutor on the conference classroom. At ITESM and at the University of Salle Bajío, coordinators of the video conference programs have found that for subjects such as mathematics, statistics and classes with "heavy contents", the presence of a tutor capable of answering students questions regarding the content of the conference makes a significant difference to the students learning and grades.

f) Tutor-student oral communication is very important, because when listening to the student, the teacher might understand some questions better orally than using other types of methods like the internet. Especially in the case of mathematics and statistics, it is very hard for students to write down some of their doubts, and this may cause problems, like using the mathematical language improperly, or overcoming technological barriers that make an extremely difficult task to write down a mathematical sign in a computer.

According to the Faculty of Education at The University of Plymouth [Plymouth], the future of videoconferencing is to incorporate video conference into web based systems, so teachers and presenters can sit in their own office or in a nearby studio and present a 'live' lecture in front of a camera attached to a web server. Using a simple switching device and several cameras, the presenter can provide remote participants with graphics, whiteboard, flipchart and other visual aids as well as alternative views of the local classroom, lecture room, etc.

Online courses. By experience at ITESM, the first step to success for online mathematics and statistics courses it is to convince students about the feasibility of the project. At this institution, full online courses are offered just for graduate students. It is also very important to have a quick response to student's questions, so they feel that "there is someone supporting them on the other side of the line".

A second step is to make sure that students can manage technology properly and have all necessary means to remain on line and to send and download information, documents, graphics, etc.

On a study conducted by Karr, Weck, Sunal and Cook [Karr 2003] at the University of Alabama to analyze the effectiveness of online learning in a graduate engineering mathematics course, they divided the class into three groups: Group A (Online course only), Group B (traditional for the first two thirds of the course and traditional and online for the final third of the course, Group C (traditional on the first third, online for the second third and traditional and online for the final third of the course). On this study they found that:

a) Students perform better on the analytical portion of the course when they had used the online mode of delivery. According to the teachers and students feedback this is due to the consistency of online materials and the fact that they have to "face the problem on their own"

b) Students taking the class by traditional mode perform better on the in-class portions of examinations. This might have been because of the instructor dropping inadvertently little hints about which aspects of the class might be on the test.

c) The two groups with a traditional mode segment perform better when they have access to both modes of deliver, traditional and online.

d) There was no significant difference on the overall performance of the groups.

From my personal experience and from some non formal studies conducted on high school and undergraduate courses it is reasonable to believe that similar results will be obtained for high school and undergraduate mathematics courses.

Many universities as ITESM consider, even for traditional courses, that online sections and online materials should be included to make courses more attractive to students and to enhance the student's performance, especially in traditionally difficult courses as mathematics.

Online problems and materials. Online problems are widely used to improve students learning on mathematics courses. Within the experience of Professor Maritza Sirvent and me, some advantages of using online problems on web based programs for mathematics problems are:

a) The bank of problems is large and includes a big variety of questions.

b) The students know immediately if their answer is correct, so they get engaged and they try the problem as many times as necessary to get the right answer.

c) Some students feel that using the computer helps them in their homework.

d) It is clear that the correctness of the problem is independent of the procedure used on the resolution. So they try their own ideas to solve the problem and use techniques as approximations using calculators. After that they study a method that will work at different situations.

e) Problems solved for students at the same class are similar but not the same so they can't copy the homework from a classmate.

f) Student's attitude toward mathematics problems seems to improve.

A disadvantage of online problems might be that, when entering the answer to a problem, sometimes the student makes a typing mistake or forgets some parenthesis and then gets an incorrect answer even if he has solved the problem correctly. Also students are not forced to write down the complete procedure, so when they are tested on a traditional writing test, they have no training on that.

WeBWorK is an internet based program to deliver homework to students on internet. It was designed by the University of Rochester. On a study conducted at Rutgers University to measure how effective WeBWorK was in improving learning measured by student's performance in Calculus [Weibel 2002], they divided students in two sections: Sections where WeBWorK homework was required weekly and it counts as part of the final grade, and sections where traditional written homework was required. Two thirds of calculus students were on WeBWorK sections, and they found the following:

a) Students in WeBWorK section did slightly better than students on traditional section. However, within WeBWorK sections, students who did over 80% of the WeBWorK problems performed dramatically better (by a full letter grade) than those who did less than half of the WeBWorK problems.

b) First year calculus students were very responsive to WeBWorK and most of them attempted every problem. They found that there is a 2-letter grade difference (on the average, from B to D) between students who do well on WeBWorK and those who do not attempt it. For upper class students taking calculus there is a 3-letter grade difference (on the average, from B to F) between students who do well on WeBWorK and those who do not attempt it. These upper class students are not very responsive to WeBWorK.

c) Students repeating calculus are not responsive to WeBWorK, and there is no significant difference on grades even for those that perform well on WeBWorK.

Online didactical material helps students to understand some concepts that might be difficult to them. Some students express that it is easier for them to read online materials than books, because they are usually more attractive and often interactive. For them it is the perfect complement for text books.

There is a bright future for online mathematics problems and didactical material. Each year the number of teachers convinced of the effectiveness of online mathematics problems and didactical material is increasing. Internet-based methods to deliver homework to students are improving and making it easier for teachers and students, saving a considerately amount of time on grading. For instance, projects such as WebALT [WebALT] aim at using existing technology standards for representing mathematics on the web and existing linguistic technologies to produce not just online mathematics problems, but language-independent mathematical didactical material.

Problem based learning (PBL) and project oriented learning (POL). These learning methodologies have been applied from elementary school to graduate programs. It is based on the principle that learning occurs not by absorbing information but by interpreting it. These methodologies are ideal for distance learning, but require that students work in teams, an arrangement that may be very difficult for some students that prefer to work individually. With these didactical techniques, learning is generated by solving a realistic situation that requires learning new concepts and applying them to solve a problem. At some universities the full curricula is build around PBL or POL techniques, while at some other universities (as ITESM) these methodologies are mixed with traditional methods [Bolaños 2003], [Watson 2002]. PBL and POL are excellent tools to introduce students on the more difficult tasks of the syllabus. The results are excellent, as statistics show that students perform better with the concepts when introduced by PBL or POL than when introduced on traditional lectures.

On these methodologies the role of the tutor is very important. The tutor is responsible for the direction of students and to help in team conflicts. The tutor has to address the student's efforts in the right direction and make suggestions about working lines. Students communicate online with their teammates and the tutor, also the final report of all teams is placed online, and so all teams might look at the similarities and differences with the solutions of the others teams.

These are just some aspects of the big world of distance learning and were choosen because we consider that they might be applied on very different teaching environments. Distance learning will continue modifying our teaching practices.

References

[Beare 89] Beare, P. L., The Comparative Effectiveness of Videotape, Audiotape, and Telelectures in Delivering Continuing Teacher Education. *The American Journal of Distance Education* **3** (2) (1989), 57–66.

[Bolaños 2003] Bolaños, G., Problem Based Learning for great statistics learning. In *Proceedings of the Hawaii International Conference on Statistics and Related Fields*, 2003.

[Greenberg 98] Greenberg, G., Distance education technologies: Best practices for K-12 settings. *IEEE Technology and Society Magazine* (Winter) 36–40.

[Karr 2003] Karr, C., Weck, B., Sunal, D. W., and Cook, T. B., Analysis of the Effectiveness of Online Learning in a Graduate Engineering Math Course. *The Journal of Interactive Online Learning* **1** (3), 2003.

[Moore 96] Moore, M., and Kearsley, G., *Distance Education: A Systems View*. Wadsworth Publishing Company, Belmont 1996.

[Moore 97] Moore, M., and Thompson, M., The Effects of Distance Learning, Revised Edition. Technical Report ACSDE Research Monograph (Number 15), American Center for the Study of Distance Education, The Pennsylvania State University, 110 Rackley Building, University Park, PA 16802-3202, 1997.

[Pflieger 61] Pflieger, E. F., and Kelly, F. G., The National Program in the Use of Television in the Public Schools. Technical Report, Ford Foundation/FAE, 1961.

[Plymouth] University of Plymouth web page, http://www2.plymouth.ac.uk/distancelearning/vidconf.html

[Russell 97] Russell, T., The "No Significant Difference" Phenomenon. Retrieved from http://tenb.mta.ca/phenom/phenom.html, 1997.

[Watson 2002] Watson, G., "Using technology to promote Success in PBL Courses". The Technology Source, 2002.

[WebALT] WebALT web page, http://webalt.math.helsinki.fi/content/index_eng.html.

[Weibel 2002] Weibel, C., and Hirsch, L., WebWork Effectiveness in Rutgers Calculus. Retrieved October 27, 2005, from http://math.rutgers.edu/~weibel/ww.html, 2002.

[Whittington 89] Whittington, N., Is Instructional Television Educationally Effective? A Research Review. Readings in Principles of Distance Education. Pennsylvania State University, 1989.

Virtual labs in mathematics education: concepts and deployment

by *Ruedi Seiler*[1]

Background. The work field of engineers, as well as that of scientists and mathematicians, is undergoing drastical changes: as numeric software and computer-algebra-systems are capable of performing intensive and complex arithmetical calculations, other abilities, such as the fast acquisition of new knowledge and new methodologies, are growing in significance. Thus, learning and teaching methods that promote life-long, efficient and independent learning have to be conveyed.

The traditional teaching methods employed at universities are of only limited success in this respect: Teacher-centered lessons provide the essential basic knowledge, but it hardly allows for a more active approach to the subject-matter. Classical experiments, in contrast, while targeted at independent knowledge acquisition, soon stumble across limits imposed by the reality of a university: high and constantly increasing numbers of participants in a course, limited access to and inadequate equipment. In addition, the experimental approach to knowledge acquisition in "real laboratories" is by its very nature limited to certain fields of studies, while more theoretical fields, such as mathematics and theoretical physics are either completely precluded or only peripherally touched upon by the existing experimental concepts.

The deployment of new media and technology in class thus represents a turning point: *Virtual Labs* are environments based on physical labs in which computer aided experiments can be designed, created, implemented and evaluated. Experiments are implemented in the form of computer-based algorithms, representing either real tools and objects or even theoretical concepts and objects.

Such explorative learning environments can be placed at the disposal of every student and teacher, independent of time and place. In the framework of the classical experimental sciences, virtual labs are capable of complementing real laboratories by allowing the concise elaboration of the actual "phenomenon" and diminishing the influence of metrological problems. As, however, the handling of the equipment and the mentioned problems represent a vital part of the acquired competence, real laboratory experiments should not be set aside completely in the experimental disciplines. In theoretical subjects, on the other hand, these technologies make abstract phenomena visually comprehensible.

In this article, we will offer detailed requirements on Virtual Labs and describe the consequences of the implementation along the lines of a prototypical Virtual Lab for Statistical Mechanics.

Pedagogical requirements. In the following, we present a list of pedagogical requirements we demand from modern e-learning technology, especially from virtual

[1] In collaboration with Thomas Richter (TU, Berlin, thor@math.tu-berlin.de) and Sabina Jeschke (TU, Berlin, sabina@math.tu-berlin.de).

laboratories. In comparison with most other e-learning environments, though, virtual labs do not define learning goals by themselves. Rather, they put "learning spaces" at the disposal of teachers and students.

A laboratory should provide the necessary equipment – or, in the case of virtual labs, the necessary algorithms – that facilitate the independent development and testing of *problem solving strategies*, incorporating typical problems of mathematics, physics and engineering science in order to prepare the student for his or her professional life.

Laboratories offer students the unique opportunity to control their learning, without outside interference and consequently being able to make an independent decision about their learning process. We divide the support of self-directed learning into the following categories:

First of all, (Virtual) Laboratories support explorative learning by allowing their users to work independently and efficiently with the technical equipment in order to investigate interconnections independently and to build an intuitive understanding of the subject. Therefore, it is vital that Virtual Laboratories should allow and encourage unconventional approaches, options, work flows, etc.

Second, the support of different learning styles is one of the utmost features of the deployment of multimedia technologies in education, even though the first generation of e-learning technologies [1] did not yet allow individual approaches to the subject. Similarly, pre-fabricated experiments might not fit into the previous knowledge of the user, strictly limited specific environments and learning goals might not fit the individual interests, failing to motivate the user. Thus, virtual laboratories must enable the user to setup and control the experiment *freely*.

Laboratories should ideally be adaptable to different application scenarios. This includes the deployment of the same basic lab in different courses, stressing different field-specific foci on the one hand, and the use in different scenarios ranging from demonstration through practice to examination on the other hand. For that reason, a virtual laboratory should not be limited to a fixed set of experiments or aimed at the requirements of one single lecture or one specific audience; for each different target audience arise different requirements. Typical application scenarios might reach from simple demonstrational support within lectures, over experiments in the classroom teaching for training and tutorials up to self-study and deployment in research applications.

Both research and engineering achievements are increasingly the result of cooperations between distributed, separated teams. Thus, team work and team-oriented projects have to be an integral part of any modern scientific education, and thus must be actively supported by virtual laboratories as well.

Laboratories must offer appropriate interfaces that will allow the integration of or linking with standard elements as Maple or Mathematica; experimental set-ups should include these elements correspondingly, for their use and handling should be a part of the scope of learning.

Laboratory elements should be detachable from the actual lab through the application of open interfaces and thus should be reusable. Such requirements not only allow the efficient construction of new laboratories from existing elements, they also ease the integration of laboratories in more complex experiments requiring additional support from outside software components.

Consequences for the implementation. The pedagogical requirements on virtual laboratories pose various demands on the software design which we demonstrate for the laboratory VideoEasel developed at the DFG Research Center. The technical focus of this laboratory is in a first, prototype phase with application to the field of statistical mechanics and related areas. Statistical problems are here modeled through the use of cellular automata, which are well-suited to design statistical models, covering many interesting areas ranging from the Ising model, statistical image denoising, lattice-gas models to Turing completeness.

In order to be able to support different and varying deployment scenarios while imposing as few restrictions on the labs themselves, it must be possible to combine the elements of laboratory equipment flexibly and creatively. This leads to a "strictly anti-monolithic", fine-granular software design, its basic structure characterized by the tripartition into simulation and arithmetic modules implementing the mathematical modules, an interface layer that serves as link between the equimentments, that allows the free combination of the software modules into an experiment, and last, graphical user interfaces allowing to control the experimental setup conveniently.

The experiments in the lab VideoEasel are implemented as small, modular units, independent of the lab's actual core, that can be created and loaded on demand. The elementary units can be separated into two distinct classes, "automata" for the algorithmic definition of physical phenomena – e.g. the Ising model – and "measuring tools" to measure certain quantities arising within the experiment – e.g. the Free Energy. VideoEasel offers basic methods for evaluation of measurements, but does not provide any numerical tools for more complex analysis or a build-in process control for more elaborate experiments. Such functions are taken over to specialized tools by utilizing the software interfaces of the laboratory, which are here realized in the middle-ware CORBA [2]. Mappings are available to many languages, such as Java, C and Python, thus facilitating the connection to various other external tools. Presently, in addition to the native Java-interfaces, there is a Python-connection for script-control, as well as a C-implementation of a Maple-connection available.

Cooperative learning strategies in virtual laboratories imply in particular that several users from different working locations can work simultaneously on a single experiment while being well aware of the actions of their partners. Therefore, the need of designing the laboratory as a multi-part network application becomes self-evident: experiments are, for example, run on a server accessible by students.

VideoEasel follows a classical client-server approach where the students control the simulations run on the server by Java front-ends. In the most simple case – as for support of a lecture in a auditorium – server and client are run on the same computer; in cooperative learning settings, the server synchronizes more clients.

Reading the above arguments concerning the requirements in implementing a virtual laboratories drafted in the previous paragraph might create the impression of a “canonical” approach. However, most existing virtual laboratories posses a narrow technical focus on specific areas and follow a monolithic design.

The second remark concerns tutorials, user guidance and the “usability” of such laboratories: The afore-mentioned flexible granular structure of the software inevitably leads to a more complex user interface and consequently to a higher adaptation time for the teaching staff as well as the students. Problems arising from the initial contact with technical problems present a prominent “motivation killer” in e-learning. In some cases, it is not easy to find the ideal compromises; to overcome this problem, one should then provide several, separate user interfaces, as for example found in VideoEasel:

For simple demonstrational applications in lectures, a Java Applet is available that allows only minimal control of an experiment. For deployment in student groups and classroom teaching, a simple but efficient Java interface has been developed; it provides more options to influence the experiment, while keeping the complexity rather low. Additional menus allow the adjustment of all kinds of parameters within the experiment. The drawing surface, though, is very similar to the applet and mimics that of standard software tools.

A more refined and complete interface was created through the Oorange toolkit [3] – also developed at the TU Berlin – allowing the purely graphic set-up of an experiment, as well as the integration and connection to other elements through “Java Beans” [4]. The server provides templates available for existing experiments, similar to the ones for the Java interfaces; these templates are transformed client-side into a Oorange compatible XML-representation. Different from the more basic interfaces, the user has the option of changing, modifying or completing the experiment at will. This access to VideoEasel does not have the pretense of being particularly easy to navigate, as it was conceived primarily for the use in research and not in teaching or in practice. Therefore, it is acceptable to require the user to go through a reasonable adaptation phase.

Last but not least, VideoEasel is also completely controlable from within the computer algebra program Maple for applications whenever the Oorange toolkit is not able to deliver the mathematical algorithms required for research purposes. This interface uses, similar to all others, the CORBA technology to exchange data between the components.

Now, in retrospective, we analyze how the required didactic concepts are implemented within VideoEasel: the field of cellular automata is rich enough to simulate interesting physical effects, yet straightforward enough to avoid undue obstacles in easy access. The basic principle of such automata can be learned quickly and allows for the execution of interesting (and esthetically pleasing) experiments through quite basic tools. Through the integration of time-proven, well known concepts – drawing programs and measuring tools – and the choice of an appropriate interface, the user is encouraged to experiment. Comprehension of the behavior of the effect to be understood is achieved through practice in the laboratory. Explorative learning is promoted through the connection of esthetical and academical contents.

The availability of various surfaces allows us to address several user groups with very different demands on the laboratories and diverse application purposes ranging from pure demonstration to research applications.

Cooperative deployment scenarios become viable through the two-part set-up as a client/server network architecture. Thus, acquisition and research between teams geographically far separated is feasible.

Finally, CORBA-interfaces allow the docking and linking of the core laboratory with other laboratories, algebra-systems and connectors to demonstrate even more complex facts and to avoid locking the user in one single laboratory technology.

Future developments. In conclusion, we will discuss some aspects of important relevance to our original aims, which are improving university education through the use of virtual laboratories:

Virtual labs, including the presented VideoEasel, are still mostly at a prototype stage. Thus, practical experience about their deployment in e-learning environments are still rare. It has to be expected that use and evaluation will result in extensive adaptations and expansions of the existing concepts, particularly in the field of usability.

To realize the pedagogical goals as presented above, it is necessary to integrate virtual laboratories into the framework of larger virtual knowledge spaces. VideoEasel does provide a number of generic interfaces which will have to be specified in more detail. More experiences with laboratories from other fields of science and engineering are necessary to define a standardized data-exchange between different laboratories.

Finally, the virtual laboratories are becoming more and more complex to use as a direct result of the diversity of addressed learning scenarios, the desired interconnectability of different applications and the broad variety of the learning contents. To counter this effect it might be desirable to extend laboratories by "digital assistants" [5]. New concepts developed in the field of artificial intelligence in recent years have to be expanded and applied to virtual knowledge spaces and their components.

References

[1] Jeschke, S., and Kohlhase, M., and Seiler, R., eLearning-, eTeaching- & eResearch-Technologien - Chancen und Potentiale für die Mathematik. *DMV-Nachrichten*, July 2004.

[2] Scallan, T., A Corba Primer. http://www.omg.org/.

[3] Oorange: The Oorange development environment. http://www.oorange.de/.

[4] *JavaBeans*. http://java.sun.com/products/ejb/.

[5] Jeschke, S., and Richter, T., and Seiler, R., Mathematics in Virtual Knowledge Spaces: User Adaption by Intelligent Assistents. In *Proceedings of the 3rd International Conference on Multimedia and ICTs in Education*, June 7–10, 2005.

Roles for the new mathematics educators

by *Mika Seppälä*

The future is here. It is just not evenly distributed. We are living interesting times! The industrial revolution is on its way in education, publishing, and business. Ways to conserve knowledge and transfer it from generation to generation are changing. Libraries are becoming digital and classes virtual. This development opens extraordinary opportunities to those willing and capable to profit from them. It also opens possibilities to spectacular failures of which we saw many some years ago.

"Emergent technology is, by its very nature, out of control, and leads to unpredictable outcomes." This certainly applies to the current development in e-learning, including e-learning mathematics. "The Future is here. It is just not evenly distributed." Both quotes are by William Gibson.

So in order to understand what lies in the future we can simply look at what our colleagues are doing today. There is no doubt that the information network and the advanced technology are going to change the way we write, publish and teach all disciplines, including mathematics, in the future. This will happen because it is possible, and because proper usage of technology will enhance our current ways to work.

To understand how educators work in 2016, we simply need to understand which, of the currently existing ways to use information technology in education, have most potential. These are likely to emerge as general paradigms and set examples that many will follow.

Changing the educational system. Not only instruction, but the whole educational system is changing. New interdisciplinary fields are emerging at a fast pace. Largely this is due to mathematics becoming more applicable thanks to the various advanced mathematics systems like Maple, Mathematica or Matlab. It is now possible to use mathematical modeling in a fundamentally deeper way than before. This is true in practically all fields, perhaps most notably in biology and medicine.

In the past, applications of mathematics in biology or medicine have been, from the mathematical point of view, rather simple. Now more complex methods can be used. This requires expertise in mathematics, computer science, and in the subject matter to which mathematics is being applied. Hence interdisciplinary study programs have been created to educate experts capable of developing these new applications.

The new roles of mathematics educators. In the past, and in many cases even today, the teaching of mathematics has been the responsibility of instructors, and the learning that of students. At most European universities, basic mathematics courses are being taught in very large sections. A typical undergraduate calculus class may have well over 100 students. In some cases these classes have hundreds of students.

The instruction is lecturing with little or no personal interactive contact between the students and the professor. Instructors simply cannot follow the day-to-day progress of their students.

Technology can be very useful here. Using systems like Maple TA or STACK, it is possible to offer automated private instruction to students and to monitor the progress of individual students even in large classes. This will empower professors and enhance traditional contact instruction in a dramatic way.

Instruction, even in the case of large classes, becomes student centered instead of instructor centered. Professors will take responsibility of their students in a way that has not been usual in the past. The emerging new role of instructors is very similar to that of coaches. Athletes have their personal coaches, so will students as well. The future instructors work like sports coaches today assisting students to achieve goals they could not achieve on their own. Empowered with advanced learning technologies, instructors can provide individual assistance to their students in a way that was not possible earlier. Interactivity can now be provided, using the web, in a way that is likely to permanently change the way we work.

Educating new educators. The inertia of the academia resists changes and delays the necessary development. Instructors in general are not ready to change the way they work. There is also a good reason for the resistance. Moving from traditional contact instruction to computer aided learning is not easy. The data in the table below are generally accepted estimates of the efforts needed for various types of teaching.

All these forms of teaching, except lecturing and small group teaching, will require additional technical support. The large spreads in the first four items reflect the fact that experienced educators can work much faster than beginning professors. There is no spread in the table for computer aided learning and interactive video. Here also experience will eventually help, but for now there are not many instructors having extensive experience in computer aided learning.

Academic work to produce one hour of student learning ([2])

Lecturing	2–10
Small group teaching	1–10
Videotaped lectures	3–10
Authoring a text	50–100
Computer aided learning	200
Interactive video	300

Using the figures of the above table, the development of a typical one semester course will amount to over five years of full time work of the author in addition to the required technical support.

Regardless of the above, some professors are developing content for computer aided learning. They are driven by the vision of greatly improved education once the necessary content is in place and available in the same ways as books are now available to students and professors.

Metadata. Developing content for computer aided learning is very costly. Furthermore, today the materials developed by professors are mostly being used only by the authors themselves and their students. Sharing does not happen, not to speak of shared development of content. To address this problem, the European Commission is currently investing heavily into projects which enhance existing content with metadata. This metadata will make content cross border usable, and shared creation of content a real possibility. The development of metadata is likely to dramatically change the way we work. It will make the hard work to develop premium on-line content cost effective and worthwhile.

Best practices. All of the above applies really to all disciplines. Problems related to the teaching of mathematics or of sciences more generally, focus on the presentation of mathematical formulae. Almost all mathematics is being written using LaTeX today. Also these proceedings have been prepared by LaTeX.

LaTeX and TeX generate extremely high quality typesetting of scientific text. These systems produce content ready for printing and publishing in the traditional way. New LaTeX classes for producing high quality presentations have been created. Practically all mathematicians are using LaTeX.

Intelligent interactivity ([3] and [4]) requires that mathematical formulae are presented in the on-line materials so that the meaning of the formulae can be automatically understood. MathML and OpenMath make this possible. To embed mathematical formulae in a proper way to web content requires the usage of these languages. LaTeX or TeX do not support MathML or OpenMath. In spite of the fact that TeX enthusiasts are working hard to develop solutions to this problem, the use of MS Word and PowerPoint together with products like MathType often makes the content development much easier.

Searching the web one can find, for example, a variety of electronic presentations of calculus or linear algebra courses. Most of these are pdf presentations of printed materials, and are not designed to be studied from the computer monitor. The new media, the computer screen, requires a different presentation of the content than what is used on printed materials. The resolution of a printed page is much higher than the resolution of the best monitors. Hence printed pages are easier to read than computer monitors. To overcome this problem, content, for the computer screen, needs to be presented in a very condensed way. For instruction based on the computer screen, the presentation of the materials needs to follow the general design principles implemented, for example, in PowerPoint.

On-line content has many important advantages which greatly overcome the handicap that computer monitors have with respect to printed pages. These advantages include hyperlinking, live interactive and adaptive content, student performance tracking, and, most recently, multilinguality. The WebALT encoding of mathematical content uses an extension of OpenMath and is such that the content can be generated in many languages automatically. Hence the content is truly multilingual, or rather, language independent. This is a serious advantage in view of the high cost of the development of on-line content.

A case study: on-line calculus at the University of Helsinki. The lesson learned from previous experiences at Florida State University was that on-line materials should use standard tools as much as possible, not require students to install new programs, and that the illustrations of mathematics should be done so that the required technicalities are completely hidden.

With these points in mind, the development of new on-line materials for calculus was started at the University of Helsinki in the Fall of 2001. These materials consist of a collection of lectures presented by PowerPoint, a collection of PowerPoint presentations of solved problems, a collection of calculus calculators empowered by MapleNET, and a repository of problems delivered to students using Maple TA, a system for the delivery and automatic grading of homework, quizzes, and examinations.

Students reactions to these new on-line materials have been overwhelmingly positive. During the Fall of 2004, a basic course in calculus was offered, at the same time, as a fully on-line course, and as a traditional lecture/problem session course. Both courses were based on the on-line materials, and had the same exercises and examinations. For the on-line students, the examinations were the only events that took place on campus and were proctored.

The results were surprising: the on-line students fared better than the traditional students in both examinations, and the retention rate was higher among the on-line students than among the traditional students.

Automatic assessment. Systems providing automatic assessment of homework problems, quizzes and examinations have been used in lower level mathematics instruction at Florida State University with spectacular results for several years. The failure rates of precalculus courses have gone down by about 50%. This is due to students being able to practice for examinations at home so that they get immediate feed-back from the system.

Currently the most advanced automatic assessment systems are Maple TA, STACK, the forthcoming LeActive Math System and the WebALT System. Common to all of these is that they offer the possibility to create algorithmic problems which are programs that generate a different version of a problem every time the program is invoked. In addition to the others, the WebALT System will also be able to generate the problem in many languages.

The algorithmic problems really make a difference. Consider, for example, the method of partial fraction decompositions. Students of calculus will have to learn that. It is relatively simple to write a program which generates over a million different but equally hard problems of partial fraction decompositions. Hence the examination about partial fraction decompositions can be published to the students before the test! Students can take the partial fraction decomposition test as many times as they want at home, get individual feed-back including full solutions. Learning by heart is not helpful, because regardless how many times they take the same test at home, they are going to get different questions in the examinations.

Such algorithmic problems were used in instruction at Florida State University in Spring 2005. Most students reacted very positively, and used the system a lot to their benefit. Some students solved even hundreds of problems on computing limits, for example. Starting in Fall 2006, students at Florida State University are required to have a laptop computer. Then the automatic assessment systems can be used in class, and examinations can be based on the use of these systems.

Conclusions. The development on-line education in mathematics at the university level has been very slow. Administrators at national agencies and ministries in various countries see the great potential that on-line content can bring to education, but largely this potential has not been realized in mathematics and, more generally, in sciences.

This is partly due to problems that one has in the presentation of scientific content on the web. The majority of on-line materials present mathematical formulae as pictures only. This is not a satisfactory solution. One cannot use a picture as a key word in a database search.

MathML and OpenMath provide solutions to this. Commercial editors, such as MS Word and PowerPoint together with MathType, provide a convenient way to produce content in which mathematics is embedded using MathML. Authoring tools are available, robust and easy to use.

Missing synchronous interactivity has been another problem in on-line instruction. Together with the introduction of tools like Skype and the various easy-to-use conferencing systems, this problem has suddenly disappeared. Virtual on-line courses can provide more personal interaction between instructors and students than a regular class with hundreds of students attending the same lectures. This development is new, and we have not yet seen how that will change instruction. The effect is likely to be impressive, however.

To use the available technology to the maximum places large demands on instructors. They have to rethink their roles and convert themselves from lecturers to coaches. And they have to be able to use technology in a fluent way. Most instructors resist doing this mainly because the transition requires a lot of work.

The main remaining obstacle in this development is the fact that premium on-line content is expensive to produce and hard to find. Extensive funding programs, like the European Commission supported Content Enhancement Projects of eContent Plus, are likely to make a dramatic difference with respect to these remaining obstacles.

The most important lessons learned were that it is necessary to keep the use of technology as simple as possible while still providing advanced functionalities. Pretty good is good enough. For the student, everything has to work right out of the box. Technicalities have to be hidden. On-line content satisfying this criteria is going to have large and permanent value. In 2016 we cannot understand how education without the information network and its services was possible.

References

[1] Bass, H., Mathematics, mathematicians, and mathematics education, *Bull. Amer. Math. Soc.* **42** (2005), 417–430.

[2] Boettcher, Judith V., Designing for Learning. http://www.designingforlearning.info/services/writing/dlmay.htm.

[3] Caprotti, O., Seppälä, M., Xambó, S., Using Web Technologies to Teach Mathematics. In *Proceedings of SITE 2006 Conference*, Orlando, FL, March 20–24, 2006.

[4] Caprotti, O., Seppälä, M., Xambó, S., Mathematical Interactive Content: What, Why and How. To appear in *Proceedings of the 1st WebALT Conference* (held in the Technical University Eindhoven, January 5–6, 2006).

[5] Grottke, S., Jeschke, S., Natho, N., Rittau, S., Seiler, R., **mArachna**: Automated Creation of Knowledge Representations for Mathematics. To appear in *Proceedings of the 1st WebALT Conference* (held in the Technical University Eindhoven, January 5-6, 2006).

Facultat de Matemàtiques i Estadística, Universitat Politècnica de Catalunya, Barcelona, Spain
E-mail: sebastia.xambo@upc.edu

School of Education, University of Michigan, Ann Arbor, Michigan, U.S.A.
E-mail: hybass@umich.edu

Instituto Tecnológico y de Estudios Superiores de Monterrey, Monterrey, México
E-mail: gbolanos@itesm.mx

Institut für Mathematik, Technische Universität Berlin, Berlin, Germany
E-mail: seiler@math.tu-berlin.de

Department of Mathematics and Statistics, University of Helsinki
and
Department of Mathematics, Florida State University
Helsinki, Finland & Talahassee, USA
E-mail: mika.seppala@webalt.net